■ Incorporating an immune response

The Basic Model does not account for immune processes (such as CD8+ cytotoxic
T lymphocytes) controlling viral load by encountering HIV-1 infected cells and killing
them. Immune recognition occurs because HIV-1 proteins and protein fragments are
presented on the membrane of infected cells.
Here we incoporate an immune response into the crHIV-1 model.
Immune cells (Z) kill HIV-1 productively infected cells at a per cell rate κ
and kill productive I_D cells at per cell rate κ'. Immune cells also die at a rate δ_Z.
Cells infected only with crHIV-1 are not recognized by immune cells because they
do not express HIV-1 proteins that can be presented on the cell membrane.

$$\dot{T} = \lambda - d\,T - k\,V\,T - k\,V_T\,T$$

$$\dot{I}_1 = k\,V\,T - b\,I_1$$

$$\dot{I}_2 = b\,(I_1 - I_2)$$

$$\cdots$$

$$\dot{I}_m = b\,I_{m-1} - \delta_0\,I_m - \kappa\,Z\,I_m$$

$$\dot{I}_T = k\,T\,V_T - d\,I_T - k\,V\,I_T$$

$$\dot{I}_{D_1} = k\,V\,\mathrm{Tt} - b\,I_{D1}$$

$$\dot{I}_{D2} = b\,(I_{D1} - I_{D2})$$

$$\cdots$$

$$\dot{I}_{D_m} = b\,I_{Dm-1} - \delta_0'\,I_{Dm} - \kappa'\,Z\,I_{D_m}$$

$$\dot{V} = n\,\delta\,I_m + D\,n\,\delta'\,I_{D_m} - c\,V$$

$$\dot{V}_T = P^2\,D\,n\,\delta'\,I_{D_m} - c\,V_T$$

for completeness we tried a variety of functions to describe
how Z cells are activated (α)

$$\dot{Z} = \alpha\,I_m - \delta_Z\,Z$$
or
$$\dot{Z} = \alpha\,(I_m + I_{D_m}) - \delta_Z\,Z$$
or
$$\dot{Z} = \alpha\,Z\,(I_m + I_{D_m}) - \delta_Z\,Z$$
or

$$\dot{Z} = \alpha - \delta_Z Z$$

(as above we used $b = 16$, $m = 30$, α and κ were calculated so that
   steady state was the initial condition for the simulations)

None of these models affect the qualitative behavior of the
crHIV–1 gene therapy model because crHIV–1 converts
the susceptible cell population T into a different reservoir of
crHIV–1 transduced cells. Thus, the crHIV–1 effect on
HIV–1 can be roughly mimicked by decreasing λ or increasing d
in the Basic Model.